Math 115

Summer 2017

Lecture 10

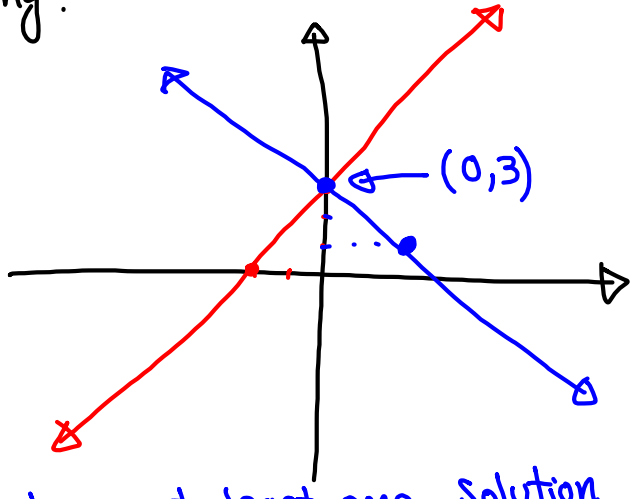
Solve by Graphing:

$$\begin{cases} 3x - 2y = -6 \\ y = \frac{-2}{3}x + 3 \end{cases}$$

Y-Int $(0,3)$

x	y
0	3
-2	0

$m = \frac{-2}{3}$



When the system has at least one solution, it is called Consistent.

When the system has exactly one solution, equations are independent.

Solve by Subs. method:

$$\begin{cases} 3x + 2y = 1 \\ x - y = -3 \end{cases}$$

$x - y = -3 \Rightarrow x = y - 3$

$3(y-3) + 2y = 1$

$3y - 9 + 2y = 1$

$5y = 10$

$y = 2$

$x = 2 - 3$

$x = -1$

Soln: $(-1, 2)$

System: Consistent

Equations: Independent

Solve by addition/elimination method:

$$\begin{cases} x - 2y = 8 \\ 3x + y = -4 \end{cases} \quad (\times 2)$$

$$\Rightarrow \begin{cases} x - 2y = 8 \\ 6x + 2y = -8 \end{cases}$$

$7x = 0 \Rightarrow x = 0$

$3(0) + y = -4$

$y = -4$

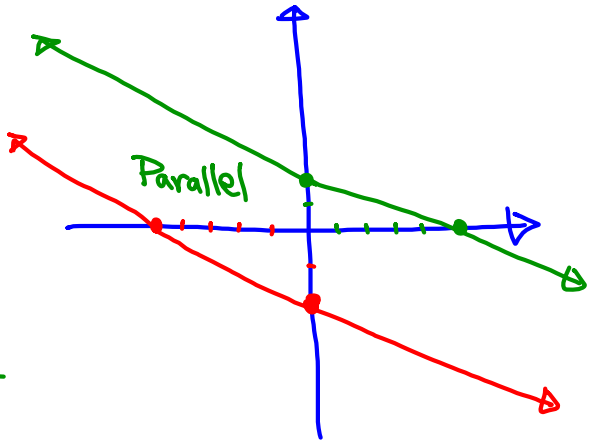
Soln: $(0, -4)$

System: Consistent

Eqns: Independent

Solve by Graphing:

$$\begin{cases} 2x + 5y = -10 \\ y = \frac{-2}{5}x + 2 \end{cases}$$



Solution: No Solution

System: Inconsistent

Equations: Independent

Solve by Subs.

$$\begin{cases} 4x - 2y = 5 \\ -2x + y = 8 \Rightarrow y = 2x + 8 \end{cases}$$

$4x - 2(2x + 8) = 5 \Rightarrow \cancel{4x} - \cancel{4x} - 16 = 5$

$-16 = 5$

false

$\emptyset$

Solution: $\emptyset$ or No Solution

System: Inconsistent

Equations: Independent

Solve by elimination / addition method:

$$\begin{cases} 3x - 2y = 15 \\ -3x + 2y = -10 \end{cases}$$

$$0 = 5$$

False

Soln: $\emptyset$, No Solution

System: Inconsistent

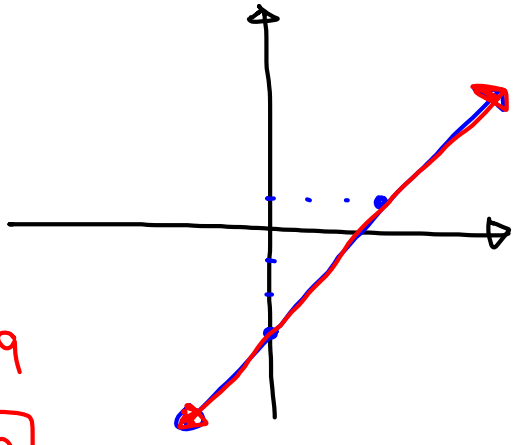
Eqns: Independent.

Solve by Graphing

$$\begin{cases} 4x - 3y = 9 \\ y = \frac{4}{3}x - 3 \end{cases}$$

$-3y = -4x + 9$

$y = \frac{4}{3}x - 3$



Soln: Infinitely Many Solutions

System: Consistent Equations: Dependent.

Solve by Subs.

$$\begin{cases} 6x - 2y = 10 \\ y = 3x - 5 \end{cases}$$

$6x - 2(3x-5) = 10$

$6x - 6x + 10 = 10$

$10 = 10$

$0 = 10 - 10$

$0 = 0$

True

Soln: Infinite # of Solns.

System: Consistents

Eqns: Dependent.

Solve by addition / elimination method:

$$4\begin{cases} 3x - y = 4 \\ 4y - 12x = -16 \end{cases} \Rightarrow \begin{cases} 12x - 4y = 16 \\ 4y - 12x = -16 \end{cases}$$

$0 = 0$

True

Soln: Infinite # of Solns.

System: Consistent

Eqns: Dependent

Solve $\begin{cases} \frac{3}{4}x + \frac{2}{3}y = 2 \\ x + \frac{y}{3} = 6 \end{cases}$

Hint: Use LCD to clear fractions

$12\left(\frac{3}{4}x + \frac{2}{3}y = 2\right) \Rightarrow 9x + 8y = 24$

$3\left(x + \frac{y}{3} = 6\right) \Rightarrow 3x + y = 18$

$-3(3x + y = 18) \Rightarrow -9x - 3y = -54$

$9x + 8y = 24$

$-9x - 3y = -54$

$5y = -30$

$y = -6$

$3x - 6 = 18$

$3x = 24$

$x = 8$

Soln: (8, -6) System: Consistent Eqns: Indep.

Solve $\begin{cases} .6x - .3y = -1.5 \\ .04x - .02y = -.1 \end{cases}$

Hint: Use powers of 10 to remove decimal point

$10(.6x - .3y = -1.5) \Rightarrow 6x - 3y = -15$

$100(.04x - .02y = -.1) \Rightarrow 4x - 2y = -10$

$\div 3$

$\div 2$

$\begin{cases} 2x - y = -5 \\ 2x - y = -5 \end{cases} \Rightarrow 0 = 0$

Infinite # of Solutions, Consistent, Dependent.

Money Problems:

Lisa has 25 coins.
Dimes & Nickels only
Total values $1.90
How many of each?

D → # of Dimes
N → # of Nickels

$$\begin{cases} D + N = 25 \\ 10D + 5N = 190 \end{cases} \quad -5$$

$$\begin{cases} -5D - 5N = -125 \\ 10D + 5N = 190 \end{cases}$$

$$5D = 65$$

$$D = 13$$

$$N = 12$$

13 Dimes
&
12 Nickels

Using one variable

Dimes → x

Nickels → $25 - x$

$$\underbrace{10x}_{\text{Dimes}} + \underbrace{5(25-x)}_{\text{Nickels}} = \underbrace{190}_{\text{Total Value}}$$

$$10x + 125 - 5x = 190$$

$$5x = 190 - 125$$

$$5x = 65$$

$$x = 13$$

13 Dimes
&
25 − 13 = 12 Nickels.

Jose has $365 in bills.

$20 bills & $5 bills only.

The number of $5 bills was 1 more than twice the number of $20 bills.

How many of each? F → $5 bills
T → $20 bills

$\div 5$ $\begin{cases} 5F + 20T = 365 \\ F = 2T + 1 \end{cases}$ $\begin{cases} F + 4T = 73 \\ F = 2T + 1 \end{cases}$

12 $20 & 25 $5

$2T + 1 + 4T = 73$

$6T = 72$ $\boxed{T = 12}$

$F = 2(12) + 1 = \boxed{25}$

Using 1-Variable

Let x be # of $20 bills

$2x+1$ ----- $5 bills

$20x + 5(2x+1) = 365$

$20x$: $20 bills Value; $5(2x+1)$: $5 bills Value; 365: Total Value

$20x + 10x + 5 = 365$

$30x = 360$

$\boxed{x = 12}$

12 $20 bills

25 $5 bills

Moe made $135 in tips. F → $5 bills
$5 bills & $10 bills only. T → $10 bills

The number of $5 bills was 5 fewer than twice the number of $10 bills.

Use system of linear equations in two variables to find how many of each?

$\div 5$ $\begin{cases} 5F + 10T = 135 \\ F = 2T - 5 \end{cases}$ $\begin{cases} F + 2T = 27 \\ F = 2T - 5 \end{cases}$

$2T - 5 + 2T = 27$

$F = 11$ $T = 8$

8 $10-bills
&
11 $5-bills

John needs 50 lb. of candy @ $1.55/lb.

He has unlimited supply of two types of candy. One @ $1.25/lb. & another one @ $1.75/lb. How many pounds of each?

$1.25	+	$1.75	=	$1.55
x lb.		y lb.		50 lb.

100 $\begin{cases} x + y = 50 \\ 1.25x + 1.75y = 1.55(50) \end{cases}$ $\div 25$ $\begin{cases} x + y = 50 \\ 125x + 175y = 155(50) \end{cases}$

$$\begin{cases} x + y = 50 \\ 5x + 7y = 310 \end{cases} \quad -5$$

$$\begin{cases} -5x - 5y = -250 \\ 5x + 7y = 310 \end{cases}$$

$$2y = 60$$

$$y = 30$$

$$x = 20$$

30 lb. of \$1.75 Type
&
20 lb. of \$1.25 Type

Larry needs 100 liters of 35% acid Solution. He has unlimited supply of 15% acid Solution & 40% acid Solution. How many liters of each Should he mix to obtain what he needs?

15% Acid	+	40% Acid	=	35% Acid
x liters		y liters		100 liters

$$\begin{cases} x + y = 100 \\ .15x + .40y = .35(100) \end{cases} \quad 100$$

$$\Rightarrow \begin{cases} x + y = 100 \\ 15x + 40y = 3500 \end{cases} \quad \div 5$$

$$-3\begin{cases} x + y = 100 \\ 3x + 8y = 700 \end{cases} \qquad \begin{cases} -3x - 3y = -300 \\ 3x + 8y = 700 \end{cases}$$

$$5y = 400$$

$$y = 80$$

80 liters of 40% acid
&
20 liters of 15% acid

Jack paid \$3.80 for 3 eggs and 4 Pancakes.

Mary paid \$2.75 for 2 eggs and 3 Pancakes. Find the price for 1 pancake.

Eliminate eggs

$$-2\begin{cases} 3E + 4P = 3.80 \\ 3\{2E + 3P = 2.75 \end{cases} \qquad \begin{cases} -6E - 8P = -7.60 \\ 6E + 9P = 8.25 \end{cases}$$

$P = .65$

\$.65 or 65¢

5 HB & 2 FF $\Rightarrow$ \$7.75

8 HB & 3 FF $\Rightarrow$ \$12.25

Find the price for 10 FF.

$$8\{5H + 2F = 7.75$$
$$-5\{8H + 3F = 12.25$$

$$\Rightarrow \begin{cases} 40H + 16F = 62 \\ -40H - 15F = 61.25 \end{cases}$$

$F = .75$

1 FF $\rightarrow$ 75 ¢ or \$.75

10 FF $\rightarrow$ 750 ¢ or \$7.50

PTA Paid \$76 to take a group of Kids & adults to the zoo.

Kid's tkt $\rightarrow$ \$3

Adult's tkt $\rightarrow$ \$8

$$\begin{cases} K + A = 17 \\ 3K + 8A = 76 \end{cases}$$

Total # of Tickets $\rightarrow$ 17

How many of each?

$K = 12$

$A = 5$

12 Kids & 5 Adults

The difference of two supplementary angles is $104°$. Find both angles.

$$\begin{cases} x + y = 180 \\ x - y = 104 \end{cases}$$

$x = 142$

$y = 38$

$142°$ & $38°$

Find two complementary angles such that the sum of 3 times one and 5 times the other one is $380°$.

$$\begin{cases} x + y = 90 \\ 3x + 5y = 380 \end{cases} \Rightarrow \begin{array}{l} x = 35 \\ y = 55 \end{array}$$

$35°$ & $55°$

Due tomorrow @ 6:00 AM

SG 11 & WP 8

You can work on WP 10.

Exam II : Monday

All exams are Cumulative.

Ch. 1, 2, 3, 8, 9, and Part of 4.

SG 1 - 13?

Ch. 4 Working with expressions, exponential rules, and operations with polynomials.

1) Distributive Prop.

$a(b+c) = ab + ac$

$a(b-c) = ab - ac$

ex: $3(2x-5) = 3 \cdot 2x - 3 \cdot 5$

$= \boxed{6x - 15}$

Distribute & Simplify

$4(2x^2 - 3x + 1) - 6(x^2 - 2x + 1)$

$= 8x^2 - 12x + 4 - 6x^2 + 12x - 6$

$= \boxed{2x^2 - 2}$

Find an expression for the perimeter in Simplest form

① $P = a + b + c$

$7x - 2$, $3x^2 - 2x + 8$, $7x^2 - 5x - 6$

$P = 7x - 2 + 7x^2 - 5x - 6 + 3x^2 - 2x + 8$

$\boxed{P = 10x^2}$

② $\boxed{P = 2L + 2W}$

$3x - 5$, $3x + 5$

$P = 2(3x + 5) + 2(3x - 5)$

$P = 6x + 10 + 6x - 10$

$\boxed{P = 12x}$

Exponential Rules:

1) x^n ← exponent

$x^n = \underbrace{x \cdot x \cdot x \cdot \dots \cdot x}_{n \text{ times}}$

x ← base

$5^3 = 5 \cdot 5 \cdot 5$, $(2x^2)^4 = 2x^2 \cdot 2x^2 \cdot 2x^2 \cdot 2x^2$

$(-3xy^3)^{10} = \underbrace{(-3xy^3) \cdot (-3xy^3) \cdot \dots \cdot (-3xy^3)}_{10 \text{ times}}$

$\left(\frac{-2}{5}\right)^5 = \frac{-2}{5} \cdot \frac{-2}{5} \cdot \frac{-2}{5} \cdot \frac{-2}{5} \cdot \frac{-2}{5}$

2) $x^1 = x$

$8^1 = 8$, $(-12x)^1 = -12x$, $(4x^2y^6)^1 = 4x^2y^6$

$\left(\frac{-5x}{12y^3}\right)^1 = \frac{-5x}{12y^3}$

3) $x^0 = 1, x \neq 0$

$10^0 = 1$, $(-100)^0 = 1$, $(4y^8)^0 = 1$, $y \neq 0$

$\left(\frac{4}{7}\right)^0 = 1$, $\left(\frac{-2x}{3y^5}\right)^0 = 1$ $x, y \neq 0$

4) $x^m \cdot x^n = x^{m+n}$

$x^3 \cdot x^7 = x^{3+7} = x^{10}$

$x^4 \cdot x^{15} \cdot x = x^{4+15+1} = x^{20}$

$(5x^3)^7 \cdot (5x^3)^8 = (5x^3)^{15}$

Find Area.

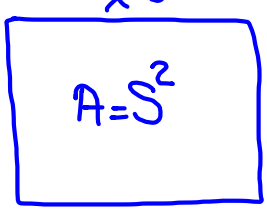
$A = LW = x^{12} \cdot x^4 = x^{16}$

5) $(x^m)^n = x^{m \cdot n}$

$(x^3)^5 = x^{3 \cdot 5} = x^{15}$

$(x^8)^2 \cdot x^4 = x^{16} \cdot x^4 = \boxed{x^{20}}$

$(y^6)^{10} \cdot (y^{10})^4 = y^{6 \cdot 10} \cdot y^{10 \cdot 4} = y^{60} \cdot y^{40} = \boxed{y^{100}}$



Find Area

$A = (x^{15})^2 = x^{30}$

6) $(xy)^n = x^n y^n$

$(2x)^4 = 2^4 x^4 = 16x^4$

$(-5x^3)^3 = (-5)^3 (x^3)^3 = \boxed{-125x^9}$

$(-2x^6y^4)^5 = (-2)^5 (x^6)^5 (y^4)^5$

$= \boxed{-32x^{30}y^{20}}$

7) $\frac{x^m}{x^n} = x^{m-n}$

$\frac{x^{10}}{x^4} = x^{10-4} = \boxed{x^6}$

$\frac{24x^{12}y^{20}}{4x^5y^{12}} = 6x^{12-5}y^{20-12}$

$= \boxed{6x^7y^8}$

$\frac{(-4x^{12})^3}{(-8x^{17})^2} = \frac{(-4)^3(x^{12})^3}{(-8)^2(x^{17})^2}$

$= \frac{-64x^{36}}{64x^{34}} = \boxed{-x^2}$

8) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \boxed{\frac{8}{27}}$

$\left(\frac{4}{x^5}\right)^2 = \frac{4^2}{(x^5)^2} = \boxed{\frac{16}{x^{10}}}$

$\left(\frac{-3x^4}{y^8}\right)^4 = \frac{(-3)^4(x^4)^4}{(y^8)^4} = \boxed{\frac{81x^{16}}{y^{32}}}$

9) $x^{-n} = \frac{1}{x^n}$

$x^{-2} = \frac{1}{x^2}$, $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

$(x^5)^{-4} = x^{-20} = \boxed{\frac{1}{x^{20}}}$

$(x^{-2})^{18} \cdot (x^{-5})^{-7} = x^{-36} \cdot x^{35} = x^{-1} = \frac{1}{x^1}$

$= \boxed{\frac{1}{x}}$

10) $\dfrac{x^{-n}}{y^{-m}} = \dfrac{y^m}{x^n}$

$\dfrac{x^{-5}}{y^{-8}} = \boxed{\dfrac{y^8}{x^5}}$

$\dfrac{x^{-2}y^7}{x^{13}y^{-3}} = \dfrac{y^3 y^7}{x^{13}x^2}$

$= \boxed{\dfrac{y^{10}}{x^{15}}}$

Class Quiz

① Find eqn of a line that contains $(-3, 2)$ with slope $\frac{1}{3}$.
Ans. in slope-Int form.

② Shade the Solution: $2x - 5y \leq 15$